S4B1: Polynomial Methods in Incidence Geometry, Harmonic Analysis and Number Theory

Unless specified otherwise, all sections referred to are from the main reference [3].

1. **Finite Field Kakeya with and without polynomials** (taken)
   Sections: 2.1, 2.2, 2.4, 3.1, 2.6.
   Remarks: If time permits, briefly touch upon Section 3.2.
   Presenter: Piro Manco

2. **The Joints Problem with and without polynomials** (taken)
   Sections: 2.5, 3.3, 3.4.
   Presenter: Tejas Ramesh

3. **Szemerédi–Trotter theorem in $\mathbb{R}^2$ using topological methods.**
   Sections: 7.1, 7.2.
   Remarks: For Section 7.2, only present a sketch of the proof of the results about crossing number of graphs.

4. **Szemerédi–Trotter Theorem in $\mathbb{R}^2$ using polynomial partitioning.** (taken)
   Sections: 10.1, 10.4, 10.2, 10.3
   Remarks: Keeping the order above in mind, first prove the Szemerédi–Trotter theorem using polynomial partitioning (Theorem 10.3). Then prove Theorem 10.3.
   Presenter: Johannes van Rensen

5. ~~**An Incidence Bound for Lines in $\mathbb{R}^3$ (Part I)**~~
   ~~**Sections:** 12.1, 12.2 till the statement of Lemma 12.6 (Planar estimate).~~
   ~~**Remarks:** Coordinate with the next presenter.~~

6. **An Incidence Bound for Lines in $\mathbb{R}^3$ ~~(Part II)~~** (taken)
   Sections: 12.2. Proof of Lemma 12.6 (Planar estimate) and Lemma 12.7 (Algebraic estimate). Wrapping up the main proof.
   Remarks: State/use the Plane Detection Lemma and Bezout theorem for lines (Theorem 6.7) without proving them.
   Presenter: Daniel Širola

7. **Interactions between Combinatorics, Geometry and Harmonic Analysis** (taken)
Sections: 15.1, 15.2 (till the statement of Theorem 15.10), 15.3.
Remarks: Depending on time constraints, 15.3.2 (proof sketch of the Hardy-Littlewood-Sobolev Inequality) can be left out.
Presenter: Izabela Mandla

8. **Oscillatory Integrals and the Kakeya problem.** (taken)
Sections: 15.4, 15.5 (till Proof of Proposition 15.26).
Presenter: Shao Liu

9. **Quantitative bounds for the Kakeya problem. Joints Theorem for Tubes using Polynomials.** (taken)
Sections: 15.6, 15.8, 15.7.
Remarks: The presentation of Section 15.7 (on the difficulties in applying the Polynomial Method to the Euclidean Kakeya problem) can be negotiated depending on time constraints.
Presenter: Dennis Wollgast

10. **Feffermann's Counter-example for the Ball Multiplier Conjecture.** (taken)
References: The original paper [1] is short and very well-written. One can also consult Section 10.1 in the textbook [2] for details.
Remarks: The second half of Section 15.5 in [3] discusses the same, but the above references might be better.
Presenter: Jianghao Zhang

11. **The Polynomial Method and Diophantine Approximation** (taken)
Sections: 16.1, 16.3, 16.4
Presenter: Edward Young

12. **Proof of Thue's Theorem**(taken)
Sections: 16.5, 16.6, 16.7, 16.8
Remarks: Can be split into two talks (with different presenters) if need be.
Presenter: Lin Feng

References

[1] Charles Fefferman, *The multiplier problem for the ball*, Annals of Mathematics **94** (1971), no. 2, 330–336.

[2] Loukas Grafakos, *Modern fourier analysis*, Vol. 250, Springer, 2009.

[3] Larry Guth, *Polynomial methods in combinatorics*, Vol. 64, American Mathematical Soc., 2016.